

(b)

$$\mathbf{A} \cdot \mathbf{B} = (2)(0) + (4)(6) + (0)(-4) = 24$$

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|} = \frac{24}{(4.47)(7.21)} = 0.745 \quad \text{or} \quad \theta = 41.9^\circ$$

- 1.6. Given  $\mathbf{F} = (y-1)\mathbf{a}_x + 2x\mathbf{a}_y$ , find the vector at  $(2, 2, 1)$  and its projection on  $\mathbf{B}$ , where  $\mathbf{B} = 5\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z$ .



$$\begin{aligned}\mathbf{F}(2, 2, 1) &= (2-1)\mathbf{a}_x + (2)(2)\mathbf{a}_y \\ &= \mathbf{a}_x + 4\mathbf{a}_y\end{aligned}$$

As indicated in Fig. 1-8, the projection of one vector on a second vector is obtained by expressing the unit vector in the direction of the second vector and taking the dot product.

$$\text{Proj. } \mathbf{A} \text{ on } \mathbf{B} = \mathbf{A} \cdot \mathbf{a}_B = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{B}|}$$

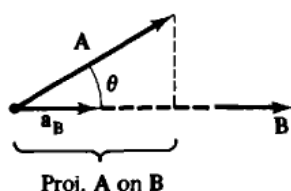


Fig. 1-8

Thus, at  $(2, 2, 1)$ ,

$$\text{Proj. } \mathbf{F} \text{ on } \mathbf{B} = \frac{\mathbf{F} \cdot \mathbf{B}}{|\mathbf{B}|} = \frac{(1)(5) + (4)(-1) + (0)(2)}{\sqrt{30}} = \frac{1}{\sqrt{30}}$$

- 1.7. Given  $\mathbf{A} = \mathbf{a}_x + \mathbf{a}_y$ ,  $\mathbf{B} = \mathbf{a}_x + 2\mathbf{a}_z$ , and  $\mathbf{C} = 2\mathbf{a}_y + \mathbf{a}_z$ , find  $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$  and compare it with  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ .

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{vmatrix} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$$

Then

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 2 & -2 & -1 \\ 0 & 2 & 1 \end{vmatrix} = -2\mathbf{a}_y + 4\mathbf{a}_z$$

A similar calculation gives  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = 2\mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ . Thus the parentheses that indicate which cross product is to be taken first are essential in the vector triple product.

- 1.8. Using the vectors  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  of Problem 1.7, find  $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$  and compare it with  $\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$ .

From Problem 1.7,  $\mathbf{B} \times \mathbf{C} = -4\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z$ . Then

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = (1)(-4) + (1)(-1) + (0)(2) = -5$$

Also from Problem 1.7,  $\mathbf{A} \times \mathbf{B} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$ . Then

$$\mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = (2)(0) + (-2)(2) + (-1)(1) = -5$$

Parentheses are not needed in the scalar triple product since it has meaning only when the cross product is taken first. In general, it can be shown that

$$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

As long as the vectors appear in the same cyclic order the result is the same. The scalar triple products not in this cyclic order have a change in sign.

- 1.9.** Express the unit vector which points from  $z = h$  on the  $z$  axis toward  $(r, \phi, 0)$  in cylindrical coordinates. See Fig. 1-9.

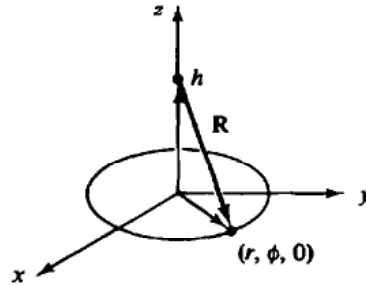


Fig. 1-9

The vector  $\mathbf{R}$  is the difference of two vectors:

$$\mathbf{R} = r\mathbf{a}_r - h\mathbf{a}_z$$

$$\mathbf{a}_R = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{r\mathbf{a}_r - h\mathbf{a}_z}{\sqrt{r^2 + h^2}}$$

The angle  $\phi$  does not appear explicitly in these expressions. Nevertheless, both  $\mathbf{R}$  and  $\mathbf{a}_R$  vary with  $\phi$  through  $\mathbf{a}_r$ .

- 1.10.** Express the unit vector which is directed toward the origin from an arbitrary point on the plane  $z = -5$ , as shown in Fig. 1-10.

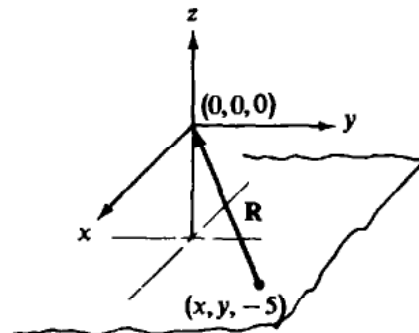


Fig. 1-10

Since the problem is in cartesian coordinates, the two-point formula of Problem 1.1 applies.

$$\mathbf{R} = -x\mathbf{a}_x - y\mathbf{a}_y + 5\mathbf{a}_z$$

$$\mathbf{a}_R = \frac{-x\mathbf{a}_x - y\mathbf{a}_y + 5\mathbf{a}_z}{\sqrt{x^2 + y^2 + 25}}$$

- 1.11.** Use the spherical coordinate system to find the area of the strip  $\alpha \leq \theta \leq \beta$  on the spherical shell of radius  $a$  (Fig. 1-11). What results when  $\alpha = 0$  and  $\beta = \pi$ ?



The differential surface element is [see Fig. 1-5(c)]

$$dS = r^2 \sin \theta \, d\theta \, d\phi$$

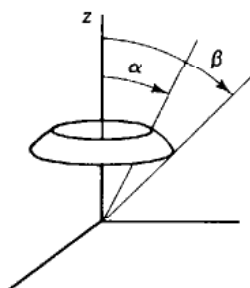


Fig. 1-11

Then

$$A = \int_0^{2\pi} \int_{\alpha}^{\beta} a^2 \sin \theta \, d\theta \, d\phi$$

$$= 2\pi a^2 (\cos \alpha - \cos \beta)$$

When  $\alpha = 0$  and  $\beta = \pi$ ,  $A = 4\pi a^2$ , the surface area of the entire sphere.

**1.12.** Obtain the expression for the volume of a sphere of radius  $a$  from the differential volume.



From Fig. 1-5(c),  $dv = r^2 \sin \theta \, dr \, d\theta \, d\phi$ . Then

$$v = \int_0^{2\pi} \int_0^{\pi} \int_0^a r^2 \sin \theta \, dr \, d\theta \, d\phi = \frac{4}{3} \pi a^3$$

**1.13.** Use the cylindrical coordinate system to find the area of the curved surface of a right circular cylinder where  $r = 2$  m,  $h = 5$  m, and  $30^\circ \leq \phi \leq 120^\circ$  (see Fig. 1-12).

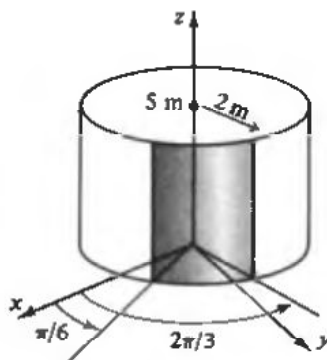


Fig. 1-12

The differential surface element is  $dS = r \, d\phi \, dz$ . Then

$$A = \int_0^5 \int_{\pi/6}^{2\pi/3} 2 \, d\phi \, dz$$

$$= 5\pi \, \text{m}^2$$

**1.14.** Transform

$$\mathbf{A} = y\mathbf{a}_x + x\mathbf{a}_y + \frac{x^2}{\sqrt{x^2 + y^2}}\mathbf{a}_z$$

from cartesian to cylindrical coordinates.